The Mathematics Enthusiast

Volume 19
Number 1 *Number 1*

Article 10

1-2022

Teacher education for integrating resources in mathematics teaching: contributions from instrumental meta-orchestration

Rosilângela Lucena

Verônica Gitirana

Luc Trouche

Teacher education for integrating resources in mathematics teaching: contributions from instrumental meta-orchestration

Rosilângela Lucena[1]
Universidade Federal de Pernambuco; Instituto Federal de Pernambuco, Brasil

Verônica Gitirana
Universidade Federal de Pernambuco, Brasil

Luc Trouche
Institut Français de l´Education, École normale supérieure de Lyon, France.

Abstract**:** This paper introduces the model of *instrumental meta-orchestration* (IMO), as an extension of the model of *instrumental orchestration*. This IMO model is defined as a systematic and intentional monitoring, by a teacher educator, of artefacts and teachers (or pre-service teachers) for facing a Meta-situation, defined as a composition of situations of different natures and difficulty levels. An IMO, in itself, is a composition of instrumental orchestrations (sequenced or interwoven). This paper presents and discusses an instrumental meta-orchestration experienced in a class of undergraduate mathematics teachers. The development of the initial model results in: new concepts, such as ad hoc reaction, didactic meta-configuration, exploitation modes and didactic meta-performance; new features, such as flexibility and interactivity; and new phenomena, such as cascade effects. It also reveals the importance of unexpected events that occur between orchestrations. Moreover, it expanded the forms of destination for evaluating specific online documents, which are named webdocs. Finally, the paper discusses the contribution to mathematics teacher education of such an extended model.

*Keywords***:** Instrumental meta-orchestration, Instrumental orchestration, Situation, Scheme, Instrumental genesis, Webdoc.

Introduction

In this paper, we present the instrumental meta-orchestration (Lucena, 2018), an extension of instrumental orchestration within the scope of teacher education and discuss its contributions to mathematics education.

Instrumental meta-orchestration (IMO) is a model for the development of teacher education approaches based on the instrumental orchestration (IO) model, introduced by Prof. Luc Trouche in 2004. Trouche (2004) developed IO to model mathematics teachers' integrattion of artefacts into teaching practice. In addition to the elements of didactic configuration and exploitation mode defined by Trouche

[1] rosi.lucenasc@gmail.com

(2004), Drijvers et al. (2010) introduce to IO the didactic performance and, in it, a look at *ad hoc* decisions, decisions taken by the teacher when unexpected events occur in the orchestration experience. Lucena (2018) assumes the importance of IO for teaching practice with digital technologies and investigates an extension of IO to mathematics teacher education model about IO, culminating in the development of instrumental meta-orchestration.

We start the paper by presenting the theoretical framework that underlies the construction of this training model, the IMO. Thus, we discuss the constitutive elements of IMO, followed by the methodology used to validate the model, with data collection methods. Some of Lucena's (2018) results are presented to discuss constructs and innovations for mathematics education, particularly for the study of instrumental orchestrations.

Instrumental Meta-orchestration Theoretical Framework

Instrumental meta-orchestration (Lucena, 2018) is a model for developing teacher training, based on instrumental orchestration (Trouche, 2004; Drijvers et al., 2010), which, by its turn, is based on the notions of situation and scheme (Vergnaud, 1964) and instrumental genesis (Rabardel, 1995) (**figure 1**).

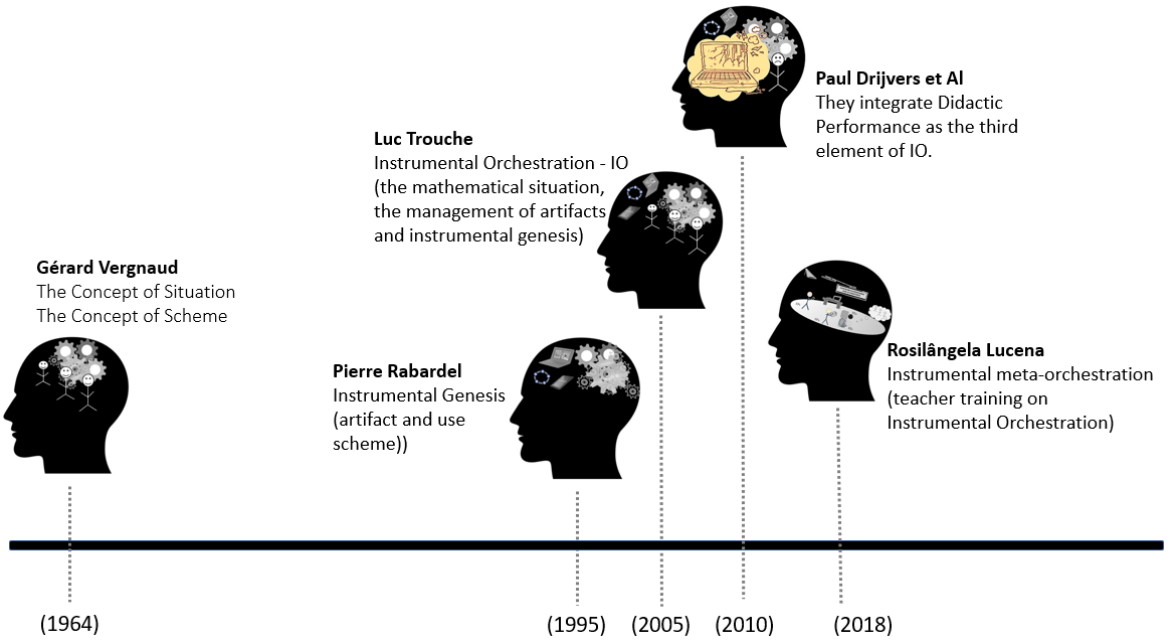

Figure 1: Theoretical framework of IMO (Adapted from Lucena, 2018, p. 34).

Situation and scheme: two essential notions

In general, a situation can be understood as the proposition of a problem aiming at developing a given mathematical knowledge. A class of situations is then a set of situations referring to the same knowledge. Vergnaud (1996) argues that an individual can evidence existing competencies or have the need to develop others when addressing a given class of situations. Competencies constitute the subject's ability to deal with different situations of the same class.

The situations give meaning to mathematical concepts, but the meaning is not in the situations themselves. Nor is it in words or mathematical symbols. The meaning is a relationship between the individual, situations, and signifiers (Vergnaud, 1996). Therefore, it is relevant to confront the individual in various situations, in the same class of situations, as this contributes to giving meaning to the concepts to which they refer.

We also adopted the notion of scheme (Vergnaud, 2009, p. 88) in the research: "a scheme is the invariant organisation of activity for a certain class of situations." This notion is hugely relevant for the understanding of what is instrumental genesis since Rabardel (1995) supported the instrumental approach development on it.

Instrumental genesis

Before defining the instrumental genesis, it is convenient to take the concept of artefact as a starting point, not with a focus on itself, but on the process of its use. For Rabardel (1995), the artefact is a human, material, and external production by one or more developers, conceived based on criteria to exercise certain functions, act as a tool, with the purposes for which it was created.

An individual who uses this artefact, develops his/her usage scheme over this usage, transforming it into his/her instrument. Thus, it is convenient to differentiate an artefact from an instrument. An instrument is a human, cognitive, and internal production of the user, which is developed individually with a collective dimension, resulting from a process called instrumental genesis. According to Rabardel, an instrument is a mixed entity formed by two components:

> - on the one hand, an artefact, material or symbolic;
> - on the other hand, one or more associated schemes of use, resulting from a specific construction of the subject, autonomous or an appropriation of ShSU [Scheme of Usage] already formed outside the subject (Rabardel, 1995, p. 95, our translation)

The development of the artifact into an instrument does not belong to the structure of the artifact, but to the schemes that the subject develops to integrate it.

Rabardel (1995) defines the instrumental genesis as the combination of two interrelated processes, instrumentation and instrumentalization:

- Instrumentation occurs when the subject inserts the artefact in his/her practice, its properties, its interface and functionalities influencing the development of schemes
- Instrumentalization occurs when the subject assigns functions to the artefact, giving rise to new ways of using it.

This relationship between the subject and the artefact, stimulated by addressing a class of situations, demands schemes already developed, to be selected from the subject's repertoire, or/and to be developed. It is the identification and analysis of these schemes in a situation that allows the researcher to make inferences about the subject's instrumental genesis. As Vergnaud (2013, p. 50) claims: "there is no situation without scheme, nor scheme without a situation." Moreover, there is no instrument without scheme; thus, there is no instrument without situations to be faced.

By studying a model for the organisation of artefacts and people to face situations and generate learning, Trouche (2004) defines instrumental orchestrations grounded on the notions of instrumental genesis and situation.

Instrumental orchestration

Instrumental Orchestration (IO) (Trouche, 2004) is a metaphor that compares the classroom to an orchestra. The conductor (the teacher) leads the musicians (his/her students) to use artefacts and transform them into musical instruments (didactic instruments), which will allow them to perform the musical sheets (solve the mathematical situations). The music results from all the procedures performed by the musicians, guided by the conductor, with their instruments, respecting the conditions imposed by the performed sheet.

> An instrumental orchestration is the systematic and intentional arrangement of the elements (artefacts and human beings) of an environment, performed by an agent (teacher) in order to effect a given situation and, in general, guide the apprentices in their instrumental genesis and the evolution and balance of their instrumental systems. (Trouche, 2005, p. 126)

An IO should be designed and analysed according to the components: didactic configuration and exploitation modes. According to Drijvers and Trouche (2008, p. 215) "[...] a didactic configuration is an arrangement of artefacts in the environment, or, in other words, a configuration of the teaching setting and the artefacts involved in it." It concerns a set of situations and didactic choices that must be made by the teacher, which will compose such configuration, such as artefacts, functions and roles and time distribution.

As regards the exploitation mode, it "is the way the teacher decides to exploit a didactical configuration for the benefit of his or her didactical intentions" (Drijvers *et al.*, 2010, p. 215). Even with roles and functions well defined in the didactic configuration, there are several ways to perform the same role. The forecast of these forms of performance is part of the exploitation mode.

To describe all the adjustments that the teacher must do during the IO experimentation, Drijvers *et al.* (2010, p. 215) introduce a third component to IO, the didactic performance, defined as follows:,

> A didactical performance involves the *ad hoc* decisions taken while teaching on how to actually perform in the chosen didactical configuration and exploitation mode: what question to pose now, how to do justice to (or to set aside) any particular student input, how to deal with an unexpected aspect of the mathematical task or the technological tool, or other emerging goals (Ibid, p. 215).

The didactic performance highlights the necessary adjustments planned, or not planned, by the teacher to meet the demands caused by the implementation of given situations in given classrooms. It also gives visibility to *ad hoc* decisions (Drijvers *et al.*, 2010) and *ad hoc* reactions (Lucena, 2018) that emerge to account for unforeseen situations and decisions taken to meet a momentary need, in order not to unsettle the whole already structured.

Ad hoc decisions are the first actions of the teacher facing unexpected phenomena in the execution of the IO, intending to fulfil didactic objectives; *ad hoc* reactions are the second, students' actions to enable

the resolution of situations proposed by the teacher, when unexpected situations occur. **Figure 2** shows the elements of the IO, as proposed by Trouche (2005), Drijvers *et al.* (2010), and Lucena (2018).

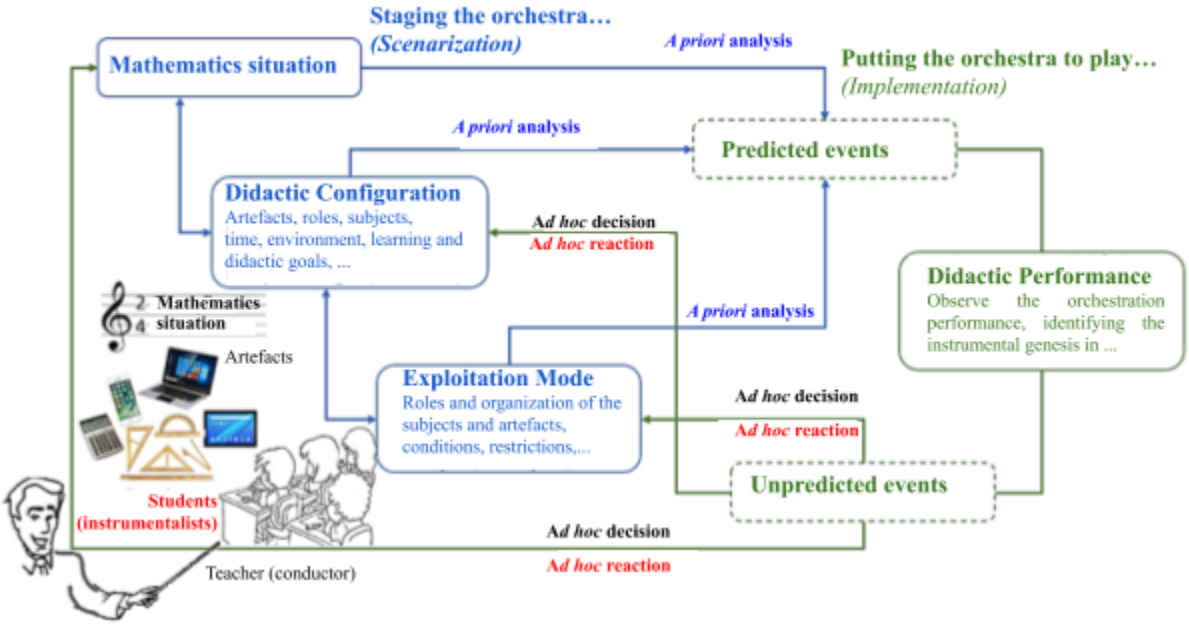

Figure 2: (Re)organisation of IO Model according to Lucena (2018, p. 34).

In **figure 2**, two stages structure the elements of an IO: the first - Scenariation - denotes the timing of the orchestration planning, the didactic configuration and the exploitation mode, to support the realisation of the situation; the second - Implementation - denotes the moment when the orchestra is put on the scene, that is, how it is performed, how it is promoting, or not, the realisation of the proposed situation - the didactic performance.

The first act, Scenariation (in blue), is orchestrated by the teacher, who determines the situation, chooses artefacts to make available, defines the roles for each subject, and the duration, and also makes the a priori analyses of didactic-pedagogical nature. The second act, Implementation (in green), is the teacher's and his/her students' experience of the first act. In it, the teacher's predictions made are confirmed, and the Implementation or not of "alternative plans" for the situations already foreseen occurs. The teacher's *ad hoc* decisions (in grey) and the students' *ad hoc* reactions (in red) to unforeseen events are also identified.

The design of an orchestration seems like such a complex process, asking us overlapping questions of mathematical, technological, and didactic natures. Bellemain and Trouche (2016) also point out teacher education as a fourth point to invest in the development of IO.

By studying an extension of the IO to mathematics teacher education about IO itself, based on a theoretical-practical-reflective model, Lucena (2018) introduces the notion of Instrumental Meta-orchestration.

Instrumental Meta-orchestration

We are going to add the prefix meta to the concepts already introduced: orchestration, situation, or even configurations, in its usual sense of a higher-level concept, as the expression metadata evokes the data relevant to the data organisation.

Instrumental meta-orchestration (Lucena, 2018) is the an agent's (teacher educator) systematic and intentional management of the artefacts and subjects (teachers and pre-service teachers) confronted with a meta-situation, to appropriate the concept Instrumental Orchestration. A meta-situation is a combination of situations aiming to encourage reflection on the Instrumental Orchestration.

Trouche (2005) underlines *systematisation*, *intentionality*, and the environment *rich in technologies* as characteristics inherent to the IO. However, considering that instrumental meta-orchestration comprises not only an orchestration but a composition of them, other characteristics emerged to account for this teacher education model (**figure 3**): the *composition* of orchestras, *sequenced or interwoven*; the *articulation* between the orchestrations; and an environment *conducive to interactions*; *adaptation* and *flexibility*.

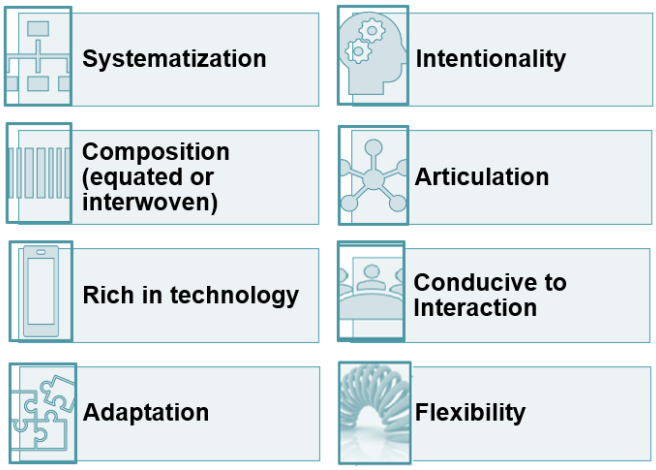

Figure 3: Characteristics of IMO (Lucena, 2018, p.135).

The systematisation aims to organise the meta-orchestration according to established parameters (as such objectives, rules, methods), individuals, artefacts and data, in a compound (sequential or interwoven) and articulated form.

Intentionality is directly related to consciously acting on the objectives that one wants to achieve. Thus, the conception, development, tests, analyses, everything proposed must be related to these objectives.

The composition of IO regards the design structure according to the proposition and its relationship with the objectives. Thus, IO can appear in a sequenced or interwoven way. In the first case, the performances of the orchestra happen at different times. In the second case, the performance of the orchestras is simultaneous. In any case, they must still be articulated with each other.

The articulation between the orchestrations promotes the (in-service or pre-service) teachers' creation of new artefacts that can provide the understanding of concepts explored in the next IO and solving the situations. Solving the situations as arranged in the IO enables that the knowledge the subjects acquired during a previous orchestration helps them both to solve the next situation and appropriate new knowledge.

The fact that instrumental orchestrations are structures rich in technologies, especially digital ones, have their essence in the central aim of IO, when created (Trouche, 2005), namely, the instrumental

genesis of the subject. It seeks to know how the subject appropriates mathematics using different devices of this nature. It does not mean despising non-digital devices. In meta-orchestration, for example, digital and analogic technologies coexist and constitute a complex system of artefacts.

As for being conducive to the subject's interaction with the artefact and between it and other subjects, different forms of work emerge. The interactions when they come from collective work, commonly, result in collaboration (they work together to reach a common goal), cooperation (each one individually performs a part of the work to reach a common goal) and in mediations (carried out by those who accompany the subjects who perform the situations).

The flexibility of the model consists not only of the possibility of making adaptations at different times of the meta-orchestration but of changing and replacing the situation and the orchestrations, always intending to improve the meta-situation and meta-orchestration. Besides, there is a possibility that these changes will be made by students as well. Sometimes, changes made by students, although not foreseen by the educators, are viable and help the orchestra success or the objectives for which the situation was created and proposed.

By adding the prefix meta, we coined those of IMO and refined. The didactic meta-configuration is an organisation of subjects (students), artefacts, didactic choices, and situations defined by the teacher (teacher educators). It presupposes, in addition to the internal organisation of each IO that will compose the IMO, the articulation and management between them. The exploitation meta-mode consists of different ways of executing the whole composition of IO and the didactic meta-configuration. At least one way of carrying out the didactic meta-configuration must be predicted.

The third component, the didactic meta-performance refers to the performance achieved by the instrumental meta-orchestration, considering the viability of the didactic meta-configuration created for the appropriation of the IO. It deals with the identification of unforeseen situations, *ad hoc* decisions (Drijvers *et al.*, 2010) and *ad hoc* reactions (Lucena, 2018) that are relevant to determine how successful the orchestrations were, both internally and externally for each orchestration.

Methodological path

The research (Lucena, 2018) was developed in two studies, one preliminary (Lucena, Gitirana, & Trouche, 2016) and one principal. This paper will discuss the implications of instrumental meta-orchestration for Mathematics Education from the results of the principal study.

The preliminary study of IMO was carried out in a master class (Lucena, Gitirana, & Trouche, 2016) and adapted for the initial mathematics teacher education class. This choice is due to the fact that the study of theoretical constructs in Mathematics Education is a standard content to the initial teacher education. It is a fertile field for courses in the appropriation of a theoretical model that promote the integration of digital technologies in teaching.

The IMO Design, part of the methodological path, refined from the results of the preliminary study, comprises the didactic meta-configuration and the exploitation mode, including theoretical analysis and event prediction, that is, Scenarisation, as well as the presentation of the research subjects and data collection structure.

Instrumental Meta-orchestration: conceiving

The IMO model (**figure 4**) is a structure that aims to assist the execution of a meta-situation, composed of six situations of different natures and complexities. The situations are related to IO learning; however, one of them aims at learning mathematics, which we call the mathematical and didactic situation with the integration of digital technologies. There is also a practical situation that aims to create an IO performance by the participants.

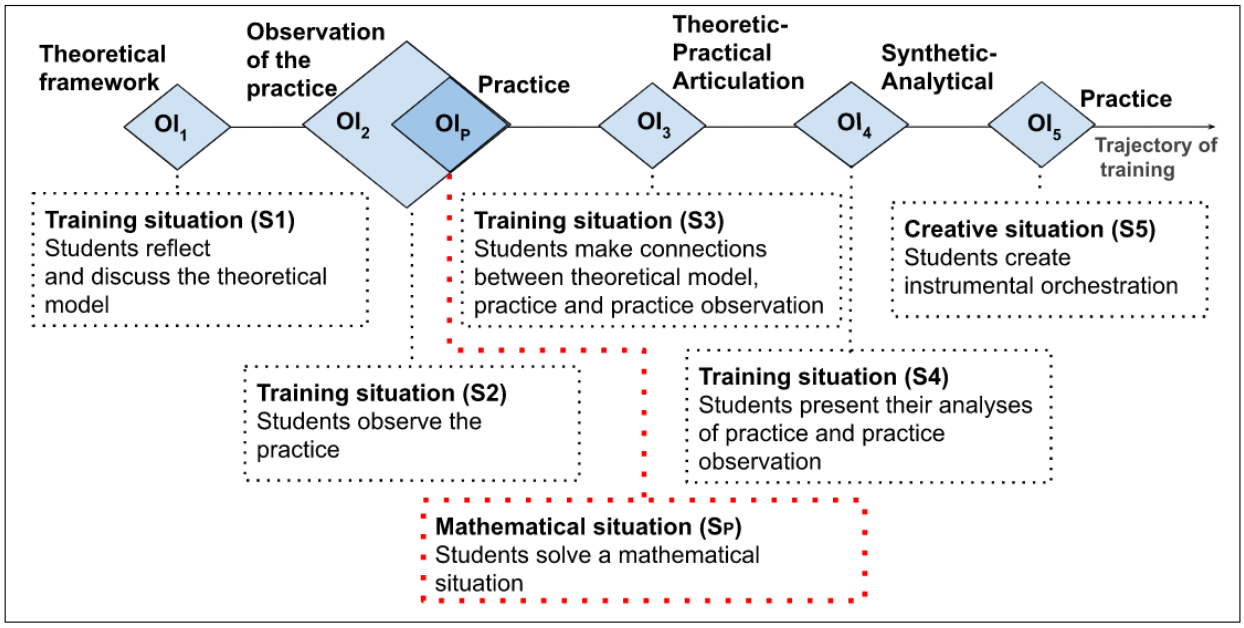

Figure 4: (Re)Design of IMO Didactic Meta-configuration (Lucena, 2018, p.135).

The objective of OI_1 is to support the theoretical background to the graduates (S_1). It is their first contact with the theoretical model (IO). This orchestration happened in a virtual environment, in distance mode.

Description of S_1 situation and its class:

a) **Training situation**: It is to introduce and discuss the theoretical model of instrumental orchestration in a forum based on reading a Webdoc (Bellemain, & Trouche, 2016). A Webdoc is defined as an interactive document available on the web, integrating a variety of data (text, images, audio, video), proposing analyses and offering means for discussing them.
b) **Class of training situation**: it is to introduce and discuss a theoretical model in a forum based on the reading of a text.

After defining S_1, the design of OI_1 intended to promote the instrumental genesis of the students regarding the webdoc use to discuss IO model in a forum. So, the design of IMO considered students' need to do S_1 in distance mode through a familiar virtual platform (Lematec Studium[2]), used in the course.

Figure 5 incorporates a trajectory expected to be followed by students and trainers when executing the IO They should perform at least two interactions during a week, through the forum after reading the

[2] *Lematec Studium*: Virtual environment to support the courses developed by Bellemain (2016).

webdoc; a first to present a question about IO; and a second to answer one of the questions asked by colleagues.

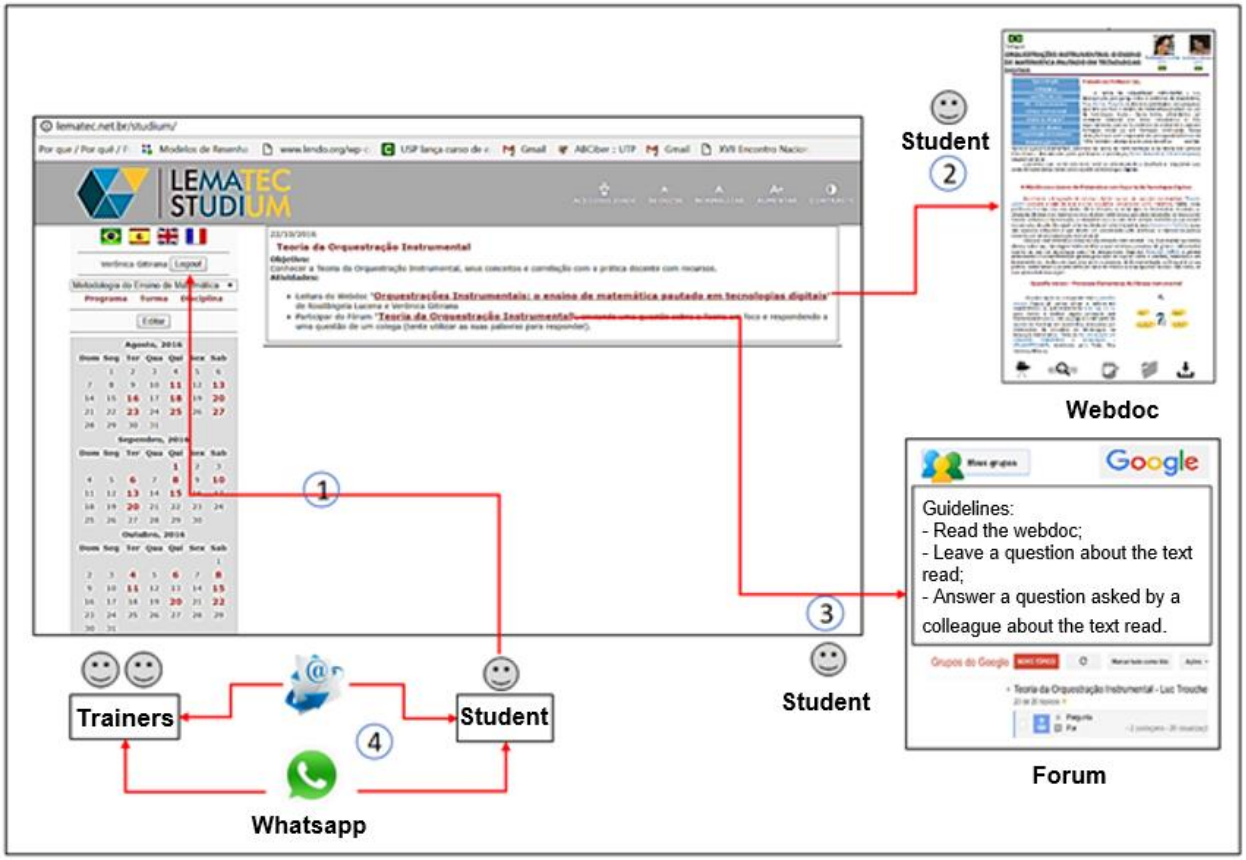

Figure 5: Design of the didactic configuration of IO_1 (Lucena, 2018, p. 142).

The interaction media used by students and teacher educators are the same as those used before in their course: e-mail and WhatsApp. Those devices were maintained because the trainers decided not to conduct mediations in the forum but stay available to provide didactic and technical support to students. The questions and answers should reveal and reflect students' doubts, understandings and misunderstandings on topics covered by the webdoc, such as artefact, instrument, situation, the notion of scheme, instrumental genesis, didactic configuration, exploitation mode and didactic performance, among others.

A webdoc, with relations between IO and examples of teaching practice with digital technologies, was developed to engage students in their reading. For that, the webdoc interface (**figure 6**), conceived

within the Lematec group (Bellemain *et al.*, 2017) from joint work with the EducTice research laboratory (Bellemain, & Trouche, 2016), was used.

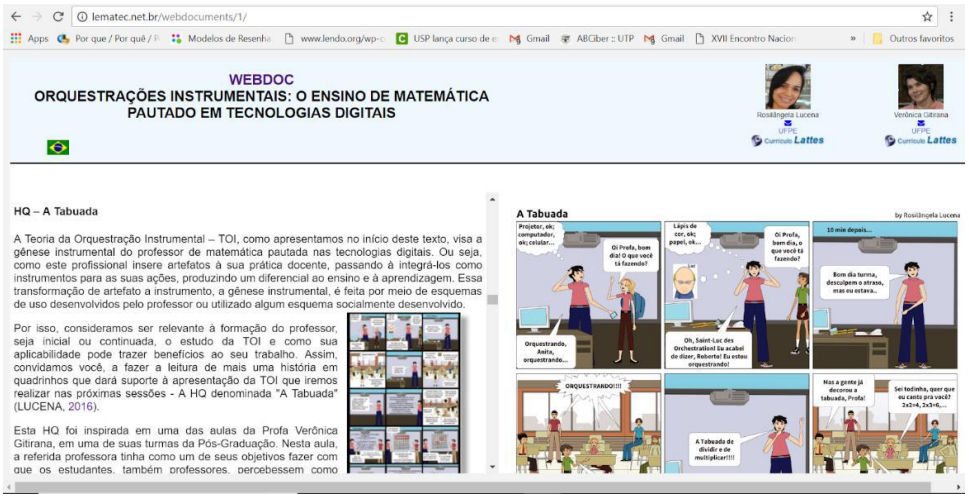

Figure 6: Partial Interface of webdoc (Lucena, 2018, p. 145).

With the webdoc, we intend to help students understand the theoretical elements that underlie IO from the articulations between theory and practice; and provide support for them to face the different situations proposed throughout the training.

So, characteristics of the webdoc form were taken into account during its construction, since its reading was one of the requirements of IMO. By investing in is the resource's form, we intended to increase the chances that students read it reflexively, a crucial factor for the success of a theoretical formation. In this sense, the authors considered the *Forms of Destination* (Remillard, 2010) as determining elements: content, structure, form of communication and publication, and the integrated media.

The webdoc content comprises three parts. The first part brings the discussion of the importance of instrumental genesis for integrating technologies in mathematics teaching. There is an intermediate section that explains this process, revealing the difference between inserting and integrating technology in the classroom (instrumentalisation/instrumentation). In the last, IO is discussed as a theoretical model to support the teacher to build instrumental orchestrations that promote instrumental genesis of students.

OI_1 should be done face-to-face; however, strikes occurred during the training period; they were decisive to decide to carry out de IO_1 itself in a distance mode. Thus, we chose to use a discussion Forum created in the G-Groups, which allows students to carry out the distance learning S_1 on days and times that are convenient, within the period determined by the trainers. The approach question-answers chose to discuss S_1, suitable for the forum, intended to induce the reading of the webdoc since to formulate questions or answer each other's questions depends on its reading, total or partial.

IOp (Pivotal Instrumental Orchestration) is the central IO, supporting the other ones (Figure 4). It is from the IOp experiment that the events that can be correlated to the theoretical elements of IO emerge and serve to relate theory and practice. It was specially developed to support participants during the realisation of the proposed mathematical situation. Unlike other IMO situations, its corresponding situation (Sp) is both mathematical and didactic:

a) **Mathematical and didactic situation**: to build a doll in GeoGebra, obeying given restrictions and resistant to changes in its shape;
b) **Class of mathematical and didactic situation**: to construct with a dynamic geometry software a geometric figure that meets certain restrictions and is resistant to changes in its shape.

From the teacher educator viewpoint, we sought to promote a practical experience of an IO solving a mathematical situation to: a) reveal the theory to support the development of teaching situations; b) know a teaching situation with the integration of software to experience instrumental genesis.

The didactic-mathematical situation aimed to exploit the concepts of proportion and symmetry; it is an elementary concept for all participants' education. Also, to do the situation, the participants should use dynamic geometry software. The chosen situation was based on Hoyles *et al.* (1991), who developed and experienced a situation of the construction of a doll in LOGO language, respecting the proportions of the human body.

Based on Laborde (1993, p. 48) discussion of the difference between drawing and figure, stressing the importance of the dragging tool in Dynamic Geometry, figures can be seen as assuming the role of reality concerning theory and of model for geometric theory. Drawing and figure are distinguished to consider this dual role. Drawing refers to the material entity, and a figure refers to the theoretical object or, as

Parzysz (1988, p. 80) puts it: "the geometric object that is described by the text that defines it." Thus, we chose to offer the puppet design in the situation (**figure 7**), so that the students kept themselves with a simple puppet, focusing on the ratios between the lengths of the puppet parts.

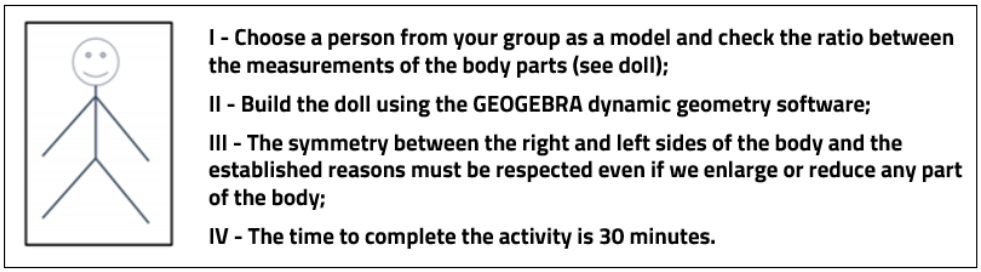

Figure 7: Mathematical didactic situation (Lucena, 2018, p. 181).

IO_2 happens simultaneously with IOp. **Figure 8** represents the scenario of the didactic configurations of both IO_2 and IOp orchestrations. In each group, two participants (duo) should assume the roles of student-actors (EA_1 and EA_2) and the others, student-observers (EO_1 and EO_2). The student-actors, also called duo, had to solve the mathematical situation in a collaborative way.

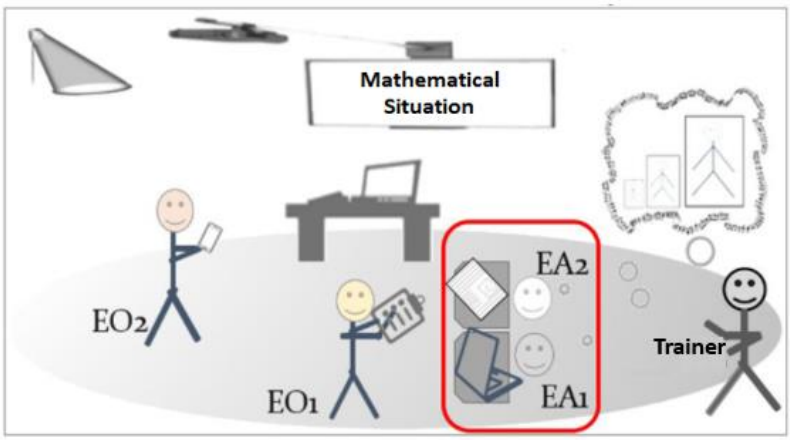

Figure 8: Scenarium of the didactic configuration of IO_2 and IOp (Lucena, 2018, p. 187).

The situation (S_2) of IO_2 places participants in the role of students-observer (EO). S_2 consists of observing the practice (Sp). EO monitors students' instrumental genesis at IOp. The IOp and IO_2 are carried out face to face in the classroom. The data produced by the EO, afterwards, could be accessed by all students at IO_4. This fact makes IOp an artefact for instrumental orchestrations.

The training situation (S_2) proposed in IO_2 and the corresponding class of training situation:

a) **Training situation**: to define criteria, in the light of the theoretical model of instrumental orchestration, and to observe IOp experiment based on these criteria;
b) **Training situation class**: to define criteria, in the light of a theoretical model, and to observe an experiment based on these criteria.

IO_2 takes place in two distinct moments: the first $IO_{2.1}$ consists of the definition of criteria to guide the observation of student-observers (EO_1 and EO_2); and the second, $IO_{2.2}$, occurs when the observation of IOp begins, simultaneously.

The training situation aims to:

A. **From the trainer's viewpoint**: To promote the appropriation of IO by observing the practice of carrying out a mathematical situation with the support of an IO.
B. **From the students' viewpoint**:
    a. To define observation criteria in $IO_{2.1}$, based on instrumental orchestration framework;
    b. To identify events by observing IOp, in $IO_{2.2}$, that highlight elements of IO model;
    c. To generate an IOp event database to support their analyses of $IO_{2.2}$ presented in IO_4.

The IO_2 training situation was created based on data collection techniques common to the research methodology and teaching practice, such as observation, logbook and filming (video and screen capture); media, day by day more common, for registering and sharing teaching work.

Thus, IO_2 didactic configuration comprises (a) the choice of two or three students to assume the role of student-observer (EO) in each group; and (b) the elaboration of a printed guide with the criteria definition, instructions about the situation to be solved and suggestions to help them executing S_2. A tablet was available to the EO students who did not want to use their own mobile to film the experiment. As for the teachers' educators, they would be responsible for presenting the situation to the students, organising and distributing the artefacts, controlling the time and providing technical support when requested.

IO_2 exploitation mode regards the collaborative work of observer-students to define observation criteria and observe EA solving the mathematical situation (IOp). The EO, during $IO_{2.2}$, should continue working individually, that is, they should not communicate with each other, nor with the other members of the group or with other colleagues in the room, except with the trainers, when necessary. EO_1 and EO_2 should focus on events that would best exemplify theoretical elements of IO, as well as guarantee the

quality of audio and image, and also register the gestures, phrases and any other aspects regarding IO elements.

IO_3 supports the situation (S_3) to articulate elements of IO and events of the practice. This orchestration, experienced at a distance in a virtual environment, is centred on the mediation of the teachers' educators. They lead students' discussion to promote articulation between students' previous experiences: reading the webdoc and the discussion already held in the forum at IO_1, the observation made at IO_2 and the experience of IOp. The trainers create a list of questions brought from the IO_1 forum. The trainers wrote others to bring up relevant elements IO, not covered by students at IO_1. The idea is to promote moments to clarify their doubts. The description of situation S_3 and its class follows:

a) **Training situation**: to discuss the IO model in a forum, according to the reading about IO, IOp experience and observation, and under participation rules;
b) **Training situation class**: to discuss the IO at a distance from different previous experiences with the model.

The participants should be able to:

A. To articulate theory and practice from reading the webdoc, IO_1 forum discussion, the IO_2 observed events and the IOp experiences as students-actors;
B. To formulate new questions about the IO model to be answered by the trainers;
C. To deepen the themes initially discussed in the first forum (IO_1) with the contributions of the participants of this forum (students, trainers and researcher).

The educators strike prevented S_3 and OI_3 from being carried out face-to-face, requiring teacher educators *ad hoc* decisions, adaptations to the training situation, and the development of a new IO. Another teacher *ad hoc* decision was to take advantage of the virtual environment used at IO_1 - the forum. Also, we sought to offer a differential to IO_3, with distance participation of the author of IO, Prof. Luc Trouche. Due to the difference of timezones between Brazil and France, five hours, the researcher's participation was asynchronous, considering that the course was nocturnal. As for the language barrier, a tool from the post-translation forum helped everyone.

Unlike the IO_1 forum, in which students had to present and answer at least one question, in the IO_3 forum, participants had ten questions posted by the trainers and answered by Prof. Luc Trouche. From

these questions and answers, students should expand the discussion, presenting new questions, answers or considerations. The ten questions (Frame 1) originate from reading all the questions and answers from students posted on the IO_1 forum. This reading helped the trainers to identify issues of interest to students focused on two themes: instrumental genesis (six questions) and instrumental orchestration (four questions). After organising the question, Prof. Luc Trouche received access to the forum. So he could answer all the questions. Subsequently, access to the forum was granted to students to start their participation.

Themes	**New questions to discuss in IO_3 forum**
Instrumental genesis	1. What is the difference between artifact and instrument? 2. What is a scheme and how can it be identified by the teacher? 3. Why should the student's instrumental genesis be promoted by the teacher? 4. What is the difference between inserting and integrating a technology into teaching practice? Does this technology have to be digital, in the context of IO? 5. What is required of the mathematics teacher to be able to integrate technologies, whether digital or not, into his teaching practice? 6. What should be considered by the mathematics teacher when following the students' instrumented actions when making the mathematical situation?
Instrumental orchestration	7. What is the role of didactic configuration, exploitation mode and didactic performance? 8. One of the actions in the didactic configuration is to predict situations that can promote success to the instrumental orchestration. Why are these predictions important? 9. What aspects reveal the success of an instrumental orchestration? And which contributions an instrumental orchestration can give to teaching practice? 10. What is the main goal of IO?

Figure 9: Questions made to IO_3 forum (Lucena, 2018, p. 279).

A didactic configuration of IO_3 has as artefacts and subjects and respective roles:

A. A virtual didactic environment to support the disciplines. Its function is to integrate the didactic configuration of IO_3 as a module of the Mathematics Teaching Methodology I course;

B. Groups google forum. The forum model chosen comprises a structure of new topics and subtopics aligned with each topic already created.

C. Two trainers: their role at IO_3 is technical and didactic support, whenever requested, and they can intervene with questions and answers at forum 3. Also, send an invitation to students to participate in the new forum and be available students via WhatsApp and e-mail to answer possible questions;

D. Students: they must access the groups in Google virtual space and read the ten questions and answers, in addition to posting on the forum, interacting with an answer or launching new questions. The interactions must be aligned with at least one of the researcher's answers, considering the themes discussed in them;

E. Internet, computers, mobiles or personal tablets: through which students interact with the webdoc and post their questions and answers;

F. Protocols: made available or generated in instrumental orchestrations before IO_3 and can be used as artefacts to carry out the proposed training situation: webdoc; posts in the first IO_1 forum;

G. Protocol and videos of the IOp observation;

H. Video and GeoGebra files of the IOp experience.

An IO_3 exploitation mode was thought, however, considering the diversity of protocols generated in the orchestrations before IO_3, we tried to show, from the didactic configuration (**figure 10**), other exploitation modes that could be adopted by students when solving the training situation.

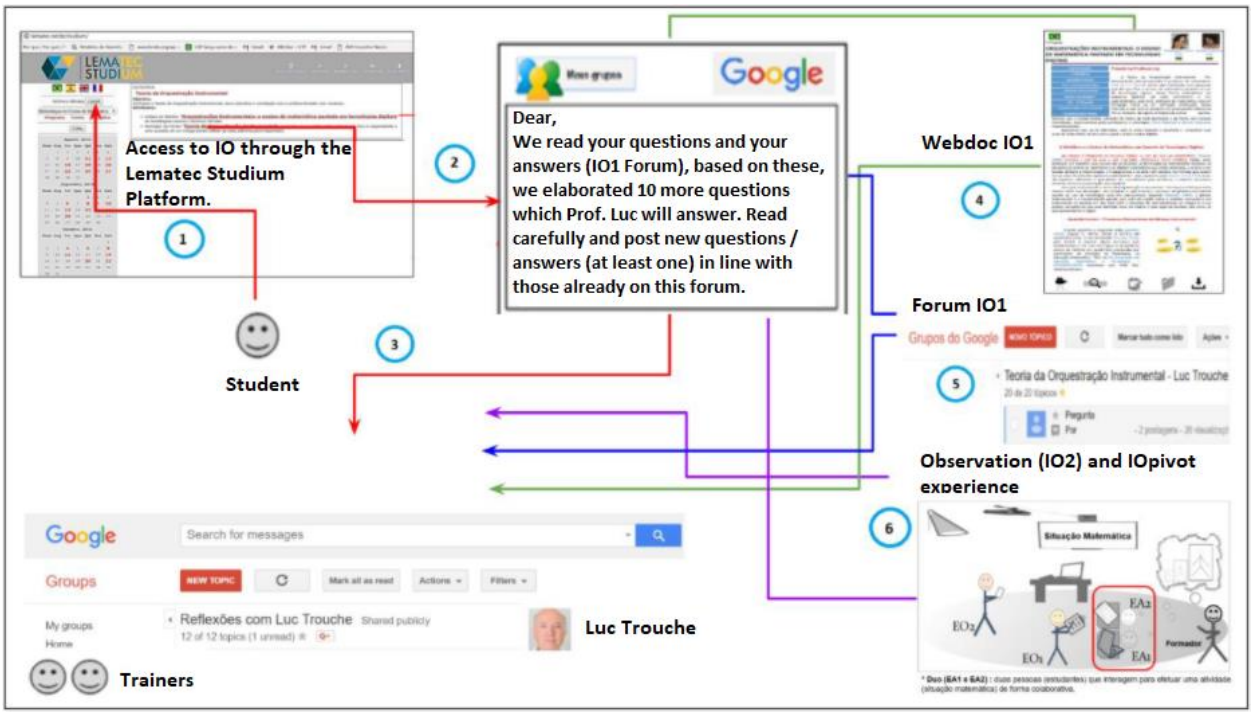

Figure 10: Design of IO_3 didactic configuration (Lucena, 2018, p. 282).

In **figure 10**, it is possible to identify the trajectory that a student participating in IO_3 can follow to solve the proposed situation. Initially, he has access through the Platform (1) to the information on how to proceed. One can also click on a direct link to the forum and webdoc. When accessing the forum (2), the student has a description of the training situation and the conditions imposed for its solution. Then, the student should read the questions posted by the trainers and the respective answers posted by Prof. Luc Trouche. From this point, the student can choose one of the ten questions/answers to expand the discussion, presenting a new question in line with the professor's answer. Then, he can locate a colleague's question and answer it on the forum (3). However, students can capture these other themes and decide to use more than the content of the researcher's answers to ask or answer.

The IO_4 intends to promote reflections, analysis and synthesis of data produced by each group (S_4). The participants present a summary of the analysis carried out for the class in a face-to-face orchestration. Thus, the situation of synthesis and analysis (S_4) is:

a) **Training situation**: to present their analysis of IOp in the light of the instrumental orchestration made to debate with the class;
b) **Training situation class**: to present their analysis of an experiment in the light of the theoretical model under study.

The resolution of the S4 training situation intends to:

A. The trainers establish links between the events experienced (IOp) and observed (IO_2) with the theoretical elements of the IO studied (IO_1 and IO_3) throughout the training;
B. The students analyse and present the analysis related to the IOp experiment in light of IO.

S_4 comprises two distinct and central sub-situations that must be carried out by each group's members: the analysis and its presentation to the class. The first sub-situation happens in group work and without virtual contact with the trainers. It is only in the presentation that the trainers could observe evidence protocols accessed and artefacts used by the students, and the theoretical elements the students were able to articulate with the experiment to proceed the analysis.

It is expected that, with support from IO_4, students will be able to:

A. To access protocols made available or those produced by them, during participation in training, to reflect on the theoretical model under study, considering practical experiences;
B. To analyse protocols data of the protocols accessed in an articulated manner with the elements of the theoretical model under study by them;
C. To organise and present data analysis with the support of one or more technologies;
D. To discuss the elements of the analysis presented based on interactions between colleagues and trainers.

A didactic configuration of IO_4 aims to promote access to the protocols of the other instrumental orchestrations already experienced favours the students' theoretical-practical reflection and, consequently, expands their understanding of the IO model.

The didactic configuration comprises: (a) artefacts - the classroom is equipped with a computer, multimedia projector and internet; (b) trainers (F1 / F2) - F1 is responsible for mediating the discussion between the group that presents the analysis and the other students in the class, after the presentation, and F2 is responsible for recording the dynamics of the classroom; (c) students - each student must contribute to their analysis, organisation and presentation. (d) protocols made available or generated in other orchestrations: webdoc; posts from IO_1 forum; cursory protocol and video of the IOp observation; video

of the duo's actions on GeoGebra, plus the audio (generated by the aTube Catcher software at IOp); GeoGebra file (history of the duo's procedures at IOp, but without audio); experience of the IOp; second forum posts (generated by students, trainers and researcher Luc Trouche at IO_3); (e) presentation time: 10 minutes for each group, plus 10 minutes to answer questions from the class and the trainers; (f) indication of formats for preparing the presentation: video (Movie Maker), comic strip (Pixton), Slide (Powerpoint, Gdrive or Prezzi presentation), timeline (MyHistro) or concept map (Cmaptools).

As exploitation mode for IO_4, Lucena (2018) predicted that each group member would, in 10 minutes, present the analysis with support from the available artefacts, sharing the presentations among participants. In the discussion time of each group presentation, their classmates assume an essential role in IO_4, as they should ask questions of the group's members based on the presentation made and articulated to the theoretical model. It is a moment of collective discussion and reflection, relevant to the participants of the training, especially in terms of deepening the theoretical elements studied. The trainers should first give opportunities for questions from their classmates.

IO_5, the last IO, integrated with the design of IMO, was conceived to support the practice of theory (S_5). In it, participants should create an IO to support a situation chosen or created by them. Among other results that may emerge from the IO_5 performance, the situation analysis and the orchestration created may reveal the students' appropriation of the IO model. It is relevant to inform that, due to the time of completion of the thesis, IO_5 was not carried out.

Data collection and analysis: its structure and subjects

The IMO, Lucena's main study (2018), was implemented in a mathematics teaching methodology class with 24 students and two trainers. The IMO composed part of the course content.

Data collection was based essentially on the observation technique, unstructured culminating in logbooks; and structured, interview and video-recording and screen-capture. The video-records emerge from a student-observer and a computer screen capture software installed in the computer used by the duo. Besides, the IMO performance generated other protocols created by the participants and, although

they are considered research results, these protocols also became artefact to following IO in the IMO, and serve data and artefact of IMO model. To analyse the IMO data, Lucena (2018) used the microgenetic-video analysis, inspired by Meira (1994). The technique favoured the organisation, classification, description and analysis of the collected data. It also contributed to create analytical instruments of the data collected with a focus on the articulation between such data and the research theoretic assumptions.

The analyses focused on: the IMO performance itself, regarding unforeseen events within an IO and between IOs; and the students' evolution during IMO performance, regarding their appropriation of the theoretical elements of IO. It is the knowledge constructed by future mathematics teachers IO, either individually or collectively.

Results: description and analysis

In this section, we will discuss the didactic meta-performance of the IMO to reveal aspects that contributed to the success of its model under development, as well as the unforeseen events, teachers' *ad hoc* decisions and students' *ad hoc* reactions taken, internally and externally to the meta-orchestration.

Instrumental meta-orchestration: design

In the implementation of the IMO, unforeseen events emerged, not only from the trainers' viewpoint trainer but also from the students' view. Thus, students react to resolve the unforeseen, *ad hoc* reaction, and the trainers take *ad hoc* decisions (Drijvers *et al.*, 2010). During the whole performance, it was possible to observe these unforeseen events (blue), *ad hoc* reactions (green) and *ad hoc* decisions (yellow) not only internally to the orchestrations, but also externally, between the performance of the orchestras that make up the meta-orchestration (**figure 11**). These can lead to changes in the situations and in the IO or require the replacement of both.

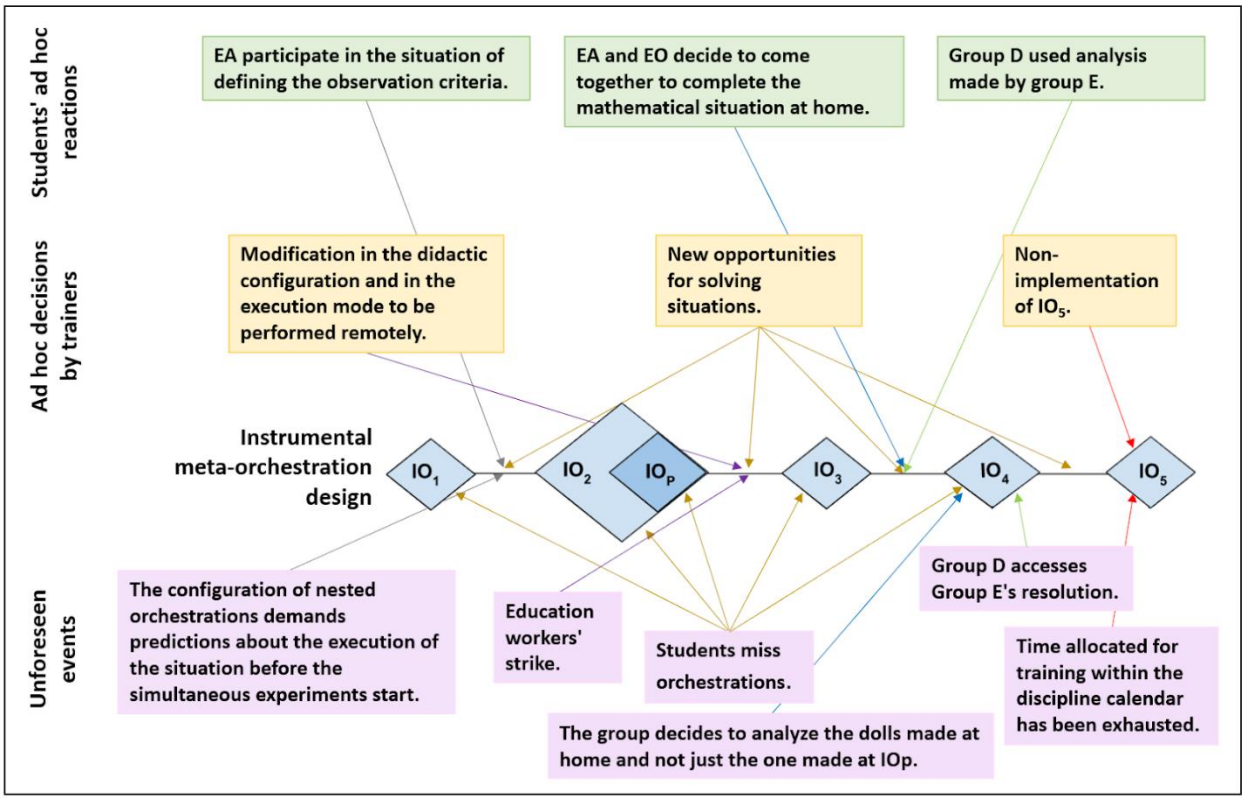

Figure 11: External events of instrumental meta-orchestration design (Lucena, 2018, p. 342).

The choice of two overlapping orchestras with simultaneous execution of IO_2 with IOp without considering that IO_2, unlike IOp, was structured to support the situation of observing the practice at two different moments (one before ($IO_{2.1}$) and the other during ($IO_{2.2}$) IOp execution) reflected directly on how the IO_2 situation should be solved. In $IO_{2.1}$, it was up to the EO decide observation criteria, without any activity planned for the EA at that moment. So, the EA students had an *ad hoc* reaction that consisted of also participating in deciding the criteria. They, then, took the opportunity to define the roles/functions of each component of the group, since the trainers did not define them. These actions occur minutes before starting $IO_{2.2}$ and IOp.

At the end of the IOp class (30 min), student-actors (EA) were unable to complete the mathematical situation. As an *ad hoc* reaction, they asked the trainers to continue solving the mathematical situation at home, the trainers authorised. The group knew that they should present an analysis of the mathematical situation solution in the IO_4, and they did not wish to analyse an unfinished solution to the situation.

Unexpectedly, the IO_4 presentations took more than one day. On the first day, some groups presented their analysis; among them, it was Group E. Their analysis of the mathematical situation, completed at home, greatly influenced Group D. Thus, the analysis presented by Group D contemplated not only the doll made by EA at IOp but also, what they (all) did at home, there was an *ad hoc* reaction.

Another unforeseen event (**figure 11**) was the Education workers (and stoppages) strike during the implementation of the IMO, making it impossible for IO_3 to occur face-to-face. Thus, the trainers took the following *ad hoc* decisions: (a) change situations S_1 and S_3 and, consequently, (b) change IO_1 and IO_3.

Something similar to what Drijvers et al. (2010) called an *ad hoc* decision between orchestrations when composing them. An unexpected event between two IO provokes a decision that is specific to the moment. Nonetheless, it has time for teachers' reflection and analysis a priori for a didactic (re)configuration.

Finally, as the IMO was implemented in a real classroom, different variables inherent to the classroom dynamics of teaching work came through. One of them is when a student misses a class. We did not foresee that the students would look for the trainers to carry out the same activities that they had missed. In this course, the teacher educator, also a trainer in this research, uses continuous evaluation. Thus, the trainers took the *ad hoc* decision to authorise the creation of new groups of students with those who had missed the class (IO_2 and IOp) using the guiding protocol. This decision guaranteed the participation of all the students in IO_4. The IO_1 and IO_3 forums were kept open so that students who did not participate by the scheduled date could also contribute. It was a strategy to allow those who missed reading the didactic webdoc.

Even not belonging to the IMO, these decisions highlight the importance of some characteristics prescribed to the model, such as articulation, systematisation, and sequencing. Even though decisions are external to the model, they work well enough to reintegrate those who have been absent from training without much prejudice.

IO_5 emerges from the iterated presentations and discussion of the design of IMO in the Research Group. It took time to understand the need for an IO in the design to offer students the opportunity to

create an IO. When IO_5 was created, the course had already ended, including the school term as well; there was no time to carry it out.

Meta-situation

The situations that compose the meta-situation have different nature and complexity. All of them play an essential role in the IMO and, when well executed, tend to promote the participants' appropriation of IO.

Unforeseen events, *ad hoc* decisions and *ad hoc* reactions internal to orchestrations, as well as the external ones, can be reflected, positively or negatively, in meta-situation or meta-orchestration. Changes and substitutions, already listed, were observed in IMO implementation. These events shed light on the complexity of meta-situation and, further, reveal that didactic meta-performance (Scenarised) comes to depend on implementation. By its turn, implementation promotes not only IO unforeseen internal events, but also between them, or even considering relationships between two IO, as is the case of the relationship between IO_2 and IOp.

Didactical meta-configuration

The didactical meta-performance observed during the implementation of the didactical meta-configuration and the exploitation mode, foreseen in the scenarisation, revealed the need for revision in its configuration and its exploitation mode.

The didactic meta-configuration scenarisation did not take into account the interval length between the implementation of two IO, but, in the case that this length is significant, it needs to be considered. Situations and sub-situations, roles and functions to be performed, during the predicted length of each IO and in the meta-orchestration (externally) to the IO, must be well defined to guarantee the effectiveness of IMO as a whole.

Another vital aspect of the didactic meta-configuration was the choice of the virtual didactic environment to implement IO_1 and IO_3. In the didactic meta-configuration, IO_1 was designed for asynchronous interaction. The same did not happen with IO_3, since it planned for face-to-face interaction.

IO_3 demanded an *ad hoc* decision from the trainers, an arrangement made before the day scheduled for the performance of this orchestra, on distance mode. The short time they had to structure the new situation, the IO and replace it with the one foreseen, justifies the difficulty of articulating the experiments carried out at IO_2 and IOp at IO_3. The students engaged well, but their posts were centred in the IO model, without much articulation with IOp.

Meta-exploitation mode

While creating the instrumental meta-configuration, the subject decides the moment each action should happen. However, it is in the exploitation mode that s/he predicts at least one way that the actors will play their roles. So, s/he can evaluate whether the predicted time will be sufficient. For example, IO_1 should take a week (remotely), IO_2 two hours, 30 minutes for IOp, a week for the new IO_3 (remotely), four hours for IO_4. The new opportunities given to the students, who missed IOp, culminated in an extension in the IMO length.

Students' IO appropriation path

The analysis carried out in each IO, separately, allowed us: to observe the students' IO appropriation path, to observe and to infer their schemes towards IO understanding; and to identify when they use a protocol they generated in an IO to solve the situation of another IO.

Lucena (2018) analyses group D appropriation path on five themes related to IO: instrumental genesis, mathematical situation, didactic configuration, exploitation mode and didactic performance. In this paper, we will discuss only the paths about the instrumental genesis and the didactic configuration. However, in the end, we bring a summary of the results present in the research.

Figure 12 shows the individual evolution of the components of group D (vertical axis), along with the IMO (horizontal axis), on dealing with the idea of instrumental genesis; the theme most discussed by the components of this group. Schemes of use, instrumentalisation, instrumentation, insertion, and integration of technologies, the distinction between instrument and artefact, are elements related to the theme that interested the group.

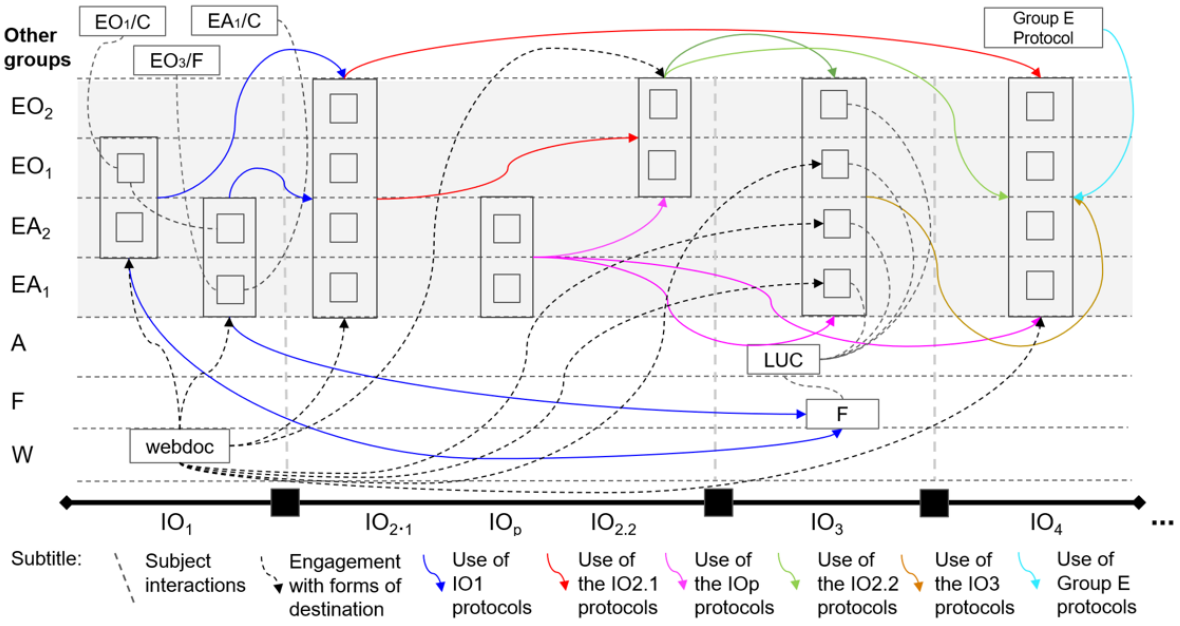

Figure 12: Instrumental genesis - thematic appropriation path (Lucena, 2018, p. 356).

Regarding the performance and evolution of individuals (**figure 12**):

- EO_2 did not interact in the IO_1, but he discussed the theme in other orchestrations. EO_2 showed advances in the appropriation of usage schemes and instrumental genesis, by reading the webdoc, evidenced in IO_2 and IO_4.
- EA_1 and EO_1 participated in all orchestrations and interacted in all of them as well. However, at IO_1, EA_1 answered a question, but she did not ask, while EO_1 asked a question but did not answer. EO_1 revealed advances in the appropriation of the notion of use schemes and instrumental genesis. As for EA_1, she revealed advances on the distinction between artefact and instrument and their relationship with instrumentalisation and instrumentation. These students' modes of engagements with the destination forms of the webdoc contributed not only to solving the situation but to understanding theoretical elements of IO.
- EA_2 participated in all orchestrations and posted a question and an answer at IO_1. He was the student who showed the most interest in this theme and sought to interact and discuss the distinction between artefact and instrument and their relationship with the instrumentalisation and

instrumentation processes. Moreover, he sought to highlight the role of schemes in such processes based on examples to make himself understood. His engagement with the destination forms of the webdoc also contributed to his understanding of such theoretical elements.

It is relevant to clarify that the notion of the scheme that the students presented was incipient. However, considering the complexity of this concept, we can say that they, mainly EA_1, were advancing towards understanding. They considered that the scheme was sometimes the actions developed by the subject, sometimes the knowledge he developed, including the previous ones. The students' understanding includes part of components of a scheme as rules of action and operative invariants. However, this was not reported to students during orchestrations, not even in the webdoc.

The didactic configuration was one of the IO components that most interested group D during the IMO (**figure 13**), especially EA_1 and EO_1, both with participation in the IO_1 and IO_3 forums. However, it was at IO_4 that the group highlights elements related to the didactic configuration. During the presentation, students point out the didactic choices of configuration made by the trainers as relevant and sometimes point out problems caused by some of these choices. In this orchestration, EO_2 did not discuss the theme in focus.

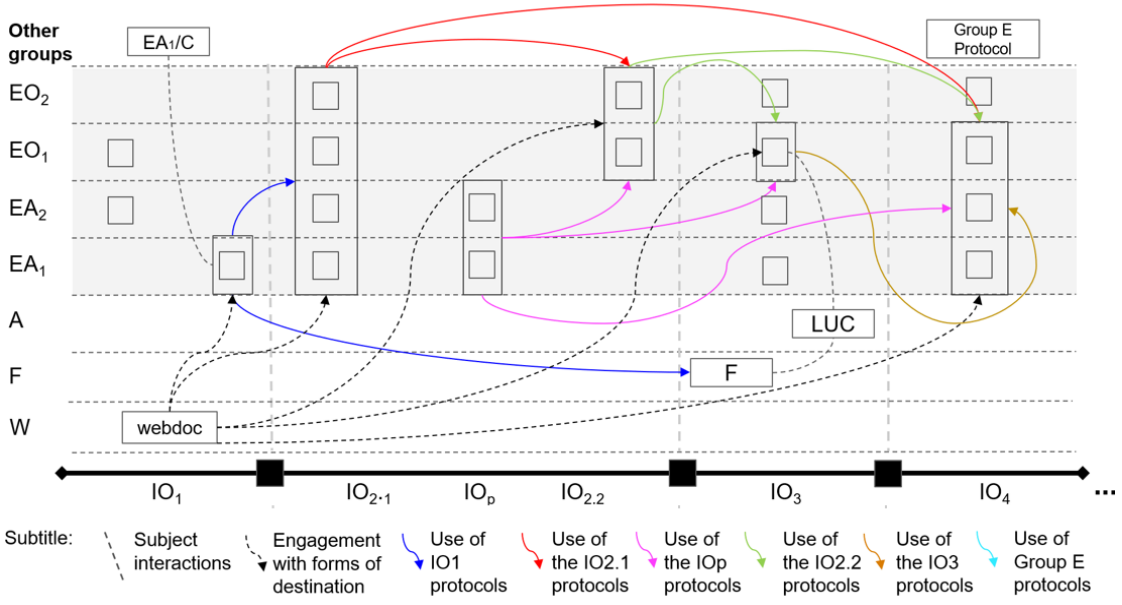

Figure 13: Didactic configuration - thematic appropriation path (Lucena, 2018, p. 361).

Regarding the individual understanding:

- EA_1 and EO_1 show greater interest in the didactic configuration than their other colleagues. They discuss this theme, bringing to light elements inherent to the configuration: the artefacts, the functions, the roles and, especially, the pedagogical time. Their understanding paths are similar regarding the choice of elements they discuss; however, from different viewpoints. EA_1 worked solving the mathematical situation; therefore, he evidenced the didactic choices of the trainer in making available, initially, the tablet and not the notebook. For EA_1, this choice did not help them solve the situation, and neither did the time they took to define a slider variable. EO_1 also discussed this fact, but as an observer. He emphasised that students-actor had experience notebooks, not with tablets. So, he attributed their fail on concluding the task in class to this choice. He argued that the roles that each actor plays are fundamental, because in their interactions, who was in control of the artefact and who collaborated favoured the partial resolution of the doll. He added that EA_1 shows a greater capacity to anticipate, and EA_2 already had some instrumentalisation with GeoGebra. EA_1 used it as a type of draft to try out specific constructions and gain insights on how to solve them.
- EA_2 also points out that the tablet made harder than helped the duo's work. However, he approved the trainers' choice. He stated that making available a different artefactthe duo was not familiar with led them to develop new use schemes to solve the situation. EA_2 also evidenced time due to the instrumentalisation/instrumentation with the tablet. The student states that the duo took a long time trying to define a variable in the tablet's slider, and this reflected in the time it took to solve the situation. Once again, EA_2 discussed elements of the IO in an articulated way and based on examples.
- EO_2 discussed elements related to the didactic configuration. However, he participated in the criteria definition process, in which time was indicated as something relevant to be observed.

Observing the students' evolution path to appropriate the theoretical elements of the IO, confirmed their interest in the themes of instrumental genesis and didactic configuration. It stands out their understanding of artefact, instrument, instrumentalisation and instrumentation, such as the use scheme. The studies with the Instrumental Orchestration carried out by Trouche (2004) and Drijvers *et al.* (2010), among others, contemplate the instrumental genesis of students to learn mathematics, sometimes of teachers to teach mathematics. Instrumental meta-orchestration expands the research perspectives from instrumental genesis to the instrumental genesis for theoretical training.

In summary, the students' appropriation paths on the theoretical elements of IO revealed:

- the webdoc as a potential artefact for teacher education, the only artefact used by group D in all the orchestrations in which they acted, except for IOp (no evidence of use was identified);
- At IO_4, the students revealed that they were able to highlight, discuss and exemplify elements of the didactic configuration, the exploitation mode and the didactic performance, but without linking them with these components of IO. For example, the variable time was discussed, without informing that it was an element of the didactic configuration;
- IOp generated an examples database, rich in events to exemplify elements of the IO. However, this protocol was not used to interact in the IO_3 forum. At the same time, it was extremely beneficial to students in their presentation at IO_4;
- External and internal events to the instrumental orchestrations of the meta-orchestration must be observed and analysed, as they can result in interferences, alterations and disturb the implementation of IMO, or even the situation.

Concluding Remarks

In a research development, it is common to have innovations that are intermediate and others that are research results. By studying the IO model, Lucena (2018) brings up an innovative element to the discussion, which are the changes brought about in this approach through the student's reaction to some proposal or fact that occurred in the classroom, that she called an *ad hoc* reaction.

Another important concept that emerged in this study, but which has been worked in parallel by other researchers, is the composition of instrumental orchestrations. The different ways orchestrations are composed and integrated; the obstacles and precautions to be taken in this integration become innovative results. The design of sequential instrumental orchestrations, to be performed in the same day, or on different days/times, especially when there are intervals between orchestrations, requires those who orchestrate to predict events that may alter or even make it impossible to perform one of the orchestras, as occurred with IO_1 and IO_3.

Vergnaud (2009) focuses on the situation as one of the pillars of a conceptual field, which is also for instrumental orchestration. Instrumental meta-orchestration deals with teachers' education; therefore, a necessary notion for its development was the discussion of a training situation with characterisation and typology. This notion is discussed aiming to allow subsidies to develop their situations and expand the types already created.

Thinking about a theoretical-practical situation for the pre-service teacher education led us to reflect on the importance of introducing theories in an accessible way to teachers at the beginning of the course. In this context, the use of webdocs, a resource built integrating different media, was crucial for presenting and retrieving different concepts that underlie IO, with articulations to practice.

Finally, the forms of destination provided by Remillard (2010) led us to think about expanding the perspectives for evaluating types of texts, when producing webdoc. The use of the webdoc raised the need to include navigation and interaction as forms of destination for this type of document.

Acknowledgements

This work was supported by grants from CAPES (Due Proc. 88881.134431/2016-01). So, we acknowledge Brazilian people who support CAPES. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect CAPES.

References

Bellemain, F. (2016). *LEMATEC Studium v. 2.0*, ambiente didático virtual. Recife-PE, LEMATEC-EDUMATEC-UFPE Research group.

Bellemain, F., Gitirana, V., Ignácio, R., Lucena, R., Tibúrcio, R., Douglas, A., Siqueira, E., & Rodrigues, A. (2017). *Webdocs Studium:* plataforma para elaboração de Webdocumentos integrando e articulando recursos dinâmicos. LEMATEC-UFPE, 2017.

Bellemain, F., & Trouche, L. (2016). Compreender o trabalho do professor com os recursos de seu ensino, um questionamento didático e informático. *Anais do I Simpósio Latino Americano de Didática de Matemática* (pp.1-32). Bonito-MS, Brazil: Universidade Federal do Mato Grosso do Sul. Retrieved from https://drive.google.com/file/d/0B6OphkgfrkD3eGRlSW1iVHg3YjQ/view.

Drijvers, P., Doorman, M., Boon, P., Reed, H., & Gravmeijer, K. (2010). The teacher and the tool: instrumental orchestrations in the technology-rich mathematics classroom. *Educational Studies in Mathematics*, 75 (2), 213-234.

Drijvers, P., & Trouche, L. (2008). From artifacts to instruments: a theoretical framework behind the orchestra metaphor. In G. W. Blume, & M. K. Heid (Eds.). *Research on technology and the teaching and learning of mathematics: cases and perspectives* (2, pp. 363-392). Charlotte, NC: Information Age.

Goigoux, R., & Vergnaud, G. (2005). Schèmes professionnels. *AIRDF*, 36(1), 7-10.

Hoyles, C., Noss, R., & Sutherland, R. (1991). *Final Report of the Microworlds Project 1986 -1989*. Department of Mathematics, Statistics and Computing. Institute of Education, University of London: Londres.

Laborde, C. (1993). The Computer as Part of the Learning Environment: the case of Geometry. In C. Keitel & K. Rutheven. *Learning from Computers: Mathematics Education and Technology* (pp. 48-67), Berlin Heidelberg: Springer Netherlands.

Lucena, R., Gitirana, V., & Trouche, L. (2016). Teoria da Orquestração Instrumental: um olhar para a formação docente. *Anais do I Simpósio Latino Americano de Didática de Matemática* (pp.1-15). Bonito-MS, Brazil: Universidade Federal do Mato Grosso do Sul.

Lucena, R. (2018). Metaorquestração instrumental: um modelo para repensar a formação de professores de matemática. (Doctoral thesis), Universidade Federal de Pernambuco, Recife, PE. Retrieved from https://repositorio.ufpe.br/handle/123456789/32844.

Meira, L. (1994). Análise microgenética e videografia: ferramentas de pesquisa em psicologia cognitiva. *Temas de Psicologia*, 2(3), 59-71.

Parzysz, B. (1988). "Knowing" vs "seeing". problems of the plane representation of space geometry figures. *Educational Studies in Mathematics*, 19(1), 79-92.

Rabardel, P. (1995). *Les hommes et les technologies: Approche cognitive des instruments contemporains*. Paris: Armand Colin.

Remillard, J. (2010). Modes d'engagement: comprendre les transactions des professeurs avec les ressources curriculaires en mathématiques. In G. Gueudet, & L. Trouche (Eds.). *Ressources vives: le travail documentaire des professeurs en mathématiques* (p. 201-216). Rennes, PUR. Lyon: INRP.

Trouche, L. (2004). Managing the complexity of human/machine interactions in computerized learning environments: guiding students' command process through instrumental orchestrations. *International Journal of Computers for Mathematical Learning*, 9, 281-307

Trouche, L. (2005). Construction et conduite des instruments dans les apprentissages mathématiques : Nécessité des orchestrations. *Recherches en Didactique des Mathématiques*, 25, 91–138.

Vergnaud, G. (1964). Essai de classification des situations d'apprentissage. *Bulletin du CERP*, 13, 145-155.

Vergnaud, G. (1996). The theory of conceptual fields. In L. P. Steffe, L. P., P. Nescher, P. Cobb, G. A. Goldin, & B. Greer (Eds.). *Theories of Mathematical learning* (p. 219-239). Mahwah, Lawrence Erlbaum Ass.

Vergnaud, G. (2009). The theory of conceptual fields. *Human development*, 52(2), 83-94.

Vergnaud, G. (2013). Pourquoi la théorie des champs conceptuels? *Infancia y Aprendizaje*, 36(2), 131-161, DOI: 10.1174/021037013806196283.